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Report Title

Wavelet approach to data analysis, manipulation, compression, and communication

ABSTRACT

The main objective of our research program over the past three-year period is three-fold: firstly, mathematical theories and methods, as well as construction of basis functions, for multi-level approximation and analysis, with emphasis on scattered data interpolation and representation, were developed; secondly, based on minimum-energy criteria, new data processing tools, particularly variational algorithms and optimal wavelet thresholding methods, with applications to image restoration, were introduced; and finally, these developments were applied to data representation, manipulation, rendering, and communication, as well as to solutions of various specific problems in computer graphics. In particular, the standard approach to subdivision schemes by constructing templates of scalar values is extended to matrix-valued subdivisions to gain flexibility and smaller template sizes for the main purpose of achieving twice continuously differentiable surfaces with more desirable geometric shapes and arbitrary topologies. As a result, one-ring templates for interpolating surface subdivisions are introduced in this project, for a wide range of applications, including reversed engineering, surface interpolation of medical and geospatial data, and visualization of point clouds. The theories and methods developed in this project have also been applied to designing algorithms for various applications in computer graphics, including adaptive stroke-based sketching with editing features, non-photorealistic graphic drawings, rendering and animation, as well as wavelet-based digital image restoration.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Journal papers published in 2005

(1) A universal noise removal algorithm with an impulse detector (with R. Garnett, T. Huegerich, W. He), IEEE Trans. Image Processing, Vol. 14, No.11 (2005), 1747--1754.

(2) Nonstationary tight wavelet frames II: unbounded intervals (with W. He and J. Stoeckler), Appl. and Comp.Harmonic Anal., Vol. 18 (2005), 25--66.

(3) Balanced multi-wavelets in R^s (with Q.T. Jiang), Math. of Computation, Vol. 74 (2005), 1323--1344.

(4) Matrix-valued symmetric templates for interpolatory surface subdivisions I: Regular vertices (with Q.T. Jiang), Appl. and Comp. Harmonic Anal., Vol. 19 (2005), 303--339.

(5) Iterative sketch generation (with W. He, H. Kang, and U. Chakraborty), The Visual Computer, Vol. 21, No. 9 (2005), 812--830.

(6) Refinable bivariate C^2-splines for multi-level data representation and surface display (with Q.T. Jiang), Math of Computation, Vol. 74 (2005), 1369--1390.

Journal papers published in 2006

(7) Refinable bivariate quartic and quintic C^2-splines for quadrilateral subdivisions (with Q.T. Jiang), J. of Comp. and Appl. Math., Vol. 196 (2006), 402--424.

(8) Affine frame decompositions and shift-invariant spaces (with Q. Sun), Appl. Comp. Harmonic Anal., Vol. 20 (2006), 74--107.

(9) Matrix-valued subdivision schemes for generating surfaces with extraordinary vertices (with Q.T. Jiang), Comp. Aided Geom. Design, Vol. 23 (2006), 419--438.

(10) Construction of orthonormal multi-wavelets with additional vanishing moments (with J. A. Lian), Adv. Comp. Math., Vol. 24 (2006), 239--262.

(11) A unified scheme for adaptive Stroke-based illustration (with H. Kang and U. Chakraborty), The Visual Computer, Vol. 22, No. 9 (2006), 814--824,

Journal papers published or awaiting publications in 2007

(12) Characterizations of tight over-sampled affine frame systems and over-sampling rates (with Q. Sun), Appl. Comp. Harmonic Anal., Vol. 22, No. 1 (2007), 1--15.

(13) Fourier transform of Bernstein-Bezier polynomials on simplices, (with T. X. He and Q.T. Jiang), J. Math. Anal. & Appl., Vol. 325 (2007), 294--304.

(14) Wavelet-based minimum-energy approach to image restoration (with J. Z. Wang), Appl. Comp. Harmonic Anal., Vol. 23, No. 1 (2007), 114--130.

(15) Triangular square-root 7 and quadrilateral square-root 5 subdivision schemes: Regular case (with Q.T. Jiang and R.D. Niang), J. Math. Anal. & Appl., Accepted for publication.

(16) From extension of Loop's approximation scheme to interpolatory subdivisions, (with Q.T. Jiang), Comp. Aided Geom. Design, Accepted for publication.

(17) Optimal Lagrange interpolation by quartic C^1 splines on triangulations (with G. Nuernberger, G. Hecklin, F. Zeilfelder), J. Comp. & Appl. Math., Accepted for publication.

(18) Coherent line drawing (with H. Kang and S. Lee), ACM SIGGRAPH on Non-photorealistic Animation and Rendering, Accepted for publication.

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Johnny Wen	0.00
FTE Equivalent:	**0.00**
Total Number:	**1**

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Total Number:	

Names of Faculty Supported

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Total Number:	

Names of Under Graduate students supported

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The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale): 0.00

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Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME
Karen Wurdack
Daba Niang
Eric Mason

Total Number: 3

Names of other research staff

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ARO Final Report

Scientific findings and accomplishments

The main objective of our research program over the past three-year period is three-fold: firstly, mathematical theories and methods, as well as construction of basis functions, for multi-level/multi-scale approximation and analysis, with emphasis on extension of the surface subdivision approach to scattered data interpolation and representation, were developed; secondly, based on certain appropriate minimum-energy criteria, new data processing tools, particularly variational algorithms and optimal wavelet thresholding methods, with applications to such applications as image restoration and point-based graphics, were introduced; and finally, these developments were applied to data representation, manipulation, compression, rendering, and communication, as well as to solutions of various specific problems in computer graphics.

The most efficient and effective approach to the endeavor of manipulating and rendering curves and surfaces in the 3-dimensional space is subdivision schemes. In fact, the subject of subdivision curves and surfaces has gained recent popularity mainly due to its great success in applications to animation movie production. According to our colleague Tony DeRose of Pixar Animation Studios, recently acquired by Walt Disney, subdivision algorithms constitute the core engine in the production of their animation movies, from the short feature film Geri's Game to the more recent successful animation movies, Finding Nemo and Incredible. Furthermore, recent adoption of the associated orthogonal, semi-orthogonal, and bi-orthogonal wavelets by the computer graphic community has added effective editing and visualization mathematical tools to the subdivision toolbox.

More recently, other industrial sectors, including the manufacturing and medical industries, have also paid attention to the development of the subdivision approach. However, resistance to adopting the subdivision approach by these industries persists for various reasons. For instance, CAD/CAM tools for the manufacturing industry often require higher orders of surface smoothness and pleasant geometric shapes for any topological structure, as well as meeting their strict requirement of error tolerance. On the other hand, for most medical and geospatial problems, as well as for applications to "reversed engineering", the subdivision surfaces are required to preserve the discrete data (or point clouds), meaning that the data used as "control vertices" are supposed to lie on the resulting subdivision surfaces. All of these requirements have created a big challenge to the computer-aided-geometric design (CAGD) community.

The biggest obstacle to the recent advancement of CAGD is that the current subdivision approach, by constructing subdivision templates of numerical values (to be called "scalar subdivisions" in this final report) has encountered at least two significant difficulties, namely: (i) design of twice continuously differentiable surfaces of arbitrary topologies in 3-D with continuous changing curvature and desirable geometric shapes is not achievable in general; and (ii) no existing interpolating scalar subdivision templates have been constructed for generating twice continuously differentiable surfaces with

relatively monotonic shapes (i.e. without unnecessary surface waviness), while preserving the discrete data (such as point clouds) in terms of surface interpolation. In fact, the problem stated in (i) for subdivision surfaces with the need of the so-called "extraordinary vertices" remains the single major open problem in CAGD, and the problem stated in (ii) has been one of the most challenging open problems to the researchers in the field.

Though we have not been able to give complete solutions to these two problem areas, we have made certain significant advancement in both directions, by extending scalars to matrices as weights, in our construction of subdivision templates. In other words, our research program on surface subdivisions departs from the standard consideration in CAGD subdivisions as described above, in that we adopt the vector approach associated with matrix-valued (as opposed to scalar-valued) subdivision templates, in achieving sufficient flexibility to accomplish the following objectives:

(1) Introduction of shape control parameters for attaining more pleasing geometric shapes of subdivision surfaces.
(2) Construction of refinable and twice continuously differentiable bivariate splines, as well as non-spline function vectors, both with minimum support, that directly give rise to the desired matrix-valued subdivision templates to achieve the property stated in (1).
(3) Construction of refinable and twice continuously differentiable interpolating bivariate splines, as well as interpolating non-spline function vectors, both with minimum support, that directly give rise to matrix-valued subdivision templates which assure interpolating surface subdivisions, in that the (initial) control vertices lie on the resulting 3-dimensional subdivision surfaces.
(4) Analytical (explicit) parametric representation of the subdivision surfaces, resulting from applying (1) and/or (2), for precise computation of design surface tolerance errors for the CAD/CAM toolbox.
(5) Construction of the corresponding wavelet tight frames with minimum support and desirable order of vanishing moments to replace the traditional (orthonormal, semi-orthogonal, and bi-orthogonal) wavelets, and to achieve analytical (explicit) representations, for designing editing tools that extend the capability of the subdivision schemes.
(6) Based on the properties of the spline representations in (2) – (5), a parametric approach for the construction of refinable function vectors that preserve the same properties as the refinable spline vectors, only with the lack of explicit representation, but with a smaller number of vector components to minimize the number of shape control parameters to facilitate implementation convenience.
(7) Construction of the matrix-valued templates for extraordinary vertices for both approximating and interpolation surface subdivisions, corresponding to (2) and (3), respectively.

In the following, we summarize some highlights of the results obtained by the support, either partially or fully, of this research grant, with references to the papers listed under "Research papers published under the sponsorship of this grant".

1. Theory of C^2 surface interpolating subdivisions [4]

As mentioned above, an obstacle to the advancement of surface subdivisions is that the current scalar approach does not allow construction of desirable interpolating subdivision schemes. Although there do exist interpolating schemes, such as the well-known butterfly scheme introduced by Dyn, Gregory, and Levin, as well as the more recent Kobbelt scheme, yet it has been experienced and generally agreed upon, however, that these interpolating schemes introduce undesirable artifacts such as surface oscillation, in general. This is due to their low smoothness order (C^1 but not C^2, even at regular vertices) and their corresponding large subdivision templates. The subdivision templates of the most recent C^2 interpolating schemes in the published literature or unpublished manuscripts are even larger, and some of them unacceptably non-symmetric. A natural route to tackle this problem is to consider vector subdivisions. However, for this approach, even the definition of interpolating surface subdivision was not understood. The correct definition should be the weakest, as long as the initial control vertices are required to lie on the resulting subdivision surfaces. This is important to allow as much flexibility as possible to design 3-dimensional interpolating C^2 surfaces. For instance, the requirement of Hermite-type interpolation considered by Dyn and Levin, as well as the definition introduced recently by Conti and Zimmermann that requires other components of the refinable function vector to be interpolating as well, are certainly too restrictive for the discovery of a general class of matrix-valued interpolating subdivision templates.

In our paper [4], we have formulated a computational effective criterion for the above-mentioned weakest definition of interpolating surface subdivisions. Using this criterion, we have constructed symmetric refinable C^2 bivariate interpolating function vectors (of vector-dimension 2 and with smallest support), by using the parametric but non-spline approach. From these examples, it is easy to formulate the corresponding symmetric 2x2 matrix-valued subdivision templates with a single ring for designing 3-dimensional interpolating C^2 surfaces. Results for both 1-to-4 split and square root 2 schemes for triangular and quadrilateral meshes are given in [4].

2. Construction of spline and non-spline C^2 surface interpolating subdivision templates [4, 6, 7]

For many applications, particularly in the CAD/CAM industry, a strict specification on subdivision-based geometric modeling requires analytical formulation of the (parametric) subdivision surface for the purpose of precise evaluation of surface normals and curvatures. In addition, interpolating subdivision is required in many applications to insure the designed vertices to lie on the resulting subdivision surface. Therefore, an interpolating spline-based approach seems to be most suitable to meet these requirements. However, the current spline approaches do not provide interpolating subdivision schemes, even for the C^1 setting. Using matrix-valued templates by means of vector

subdivisions, we have constructed in [4, 6, 7] certain C^2 bivariate spline interpolating subdivision schemes for ordinary vertices for various triangular and rectangular splits.

Specific results obtained in [6] and [7] are summarized as follows. For the 1-to-4 triangular-split surface subdivision, the support of the minimum-supported C^2 cubic bivariate spline on the type-3 triangulation (i.e. 6-directional mesh, obtained by a 1-to-6 Powell-Sabin split of each triangle in a 3-directional mesh), introduced in our earlier paper (supported by the previous ARO grant), is slightly enlarged in [6] to achieve the interpolating property while preserving symmetry. For the 1-to-4 quadrilateral-split surface subdivision, refinable C^2 quartic bivariate spline vectors on the type-2 triangulation (i.e. 4-directional mesh) are constructed in [7] to achieve the interpolating property while preserving symmetry as well. It should be mentioned that the subdivision templates for these interpolating spline schemes are slightly larger (but exceeding by much less than one additional ring) of the non-spline C^2 interpolating scheme in [4], with a single ring (i.e. minimum size), as described above.

3. Interpolating subdivision templates for extraordinary vertices [9, 16]

The single most important open problem in CAGD is the design of C^2 surfaces of arbitrary topologies with continuous change of curvature and desirable geometric shapes via the subdivision approach. This problem is frequently referred to as the Holy Grail problem of the field. Although C^2 algorithms do exist in the literature, the geometric shape of the resulting subdivision surfaces by applying these algorithms at "extraordinary vertices" are found to be very poor, being too "flat" (with zero curvature). While we are still unable to solve this problem, we have introduced the notion of "shape control parameters" via vector subdivisions for adjusting the geometric shapes at extraordinary vertices, without changing the C^2 smoothness property. We hope that this approach gives a breath of fresh air to the research progress of the Holy Grail problem.

In this direction, we have developed, in [9], certain smoothness analysis of vector surface subdivision schemes near extraordinary vertices. We have also extended the general C^k continuity criteria of Reif and of Prautzsch, for k > 1, from scalar subdivisions to vector subdivisions, in this paper. In addition, we have demonstrated with examples in [9] that the shape control parameters could indeed be used to effectively adjust geometric shapes of subdivision surfaces. In our more recent paper [16], we carry out the extension of results from [4, 6] to extraordinary vertices of arbitrary valences (also called degrees) by applying our results in [9]. This paper also extends the well-known box-spline based Loop's scheme to interpolating subdivisions, for vertices of all valences.

4. Other surface subdivision schemes and interpolation algorithms [4, 15, 17]

The papers [4, 6, 7] are devoted to the commonly used 1-to-4 triangular and quadrilateral splits, although the theoretical results in [4] cover all types of splits, including the then popular square-root 3 (split) subdivision, with an example in the [same

paper to illustrate an extension of Loop's scheme, not only from scalar subdivision to matrix subdivision to add geometric shape flexibility, but also from 1-to-4 triangular split to square-root 3 surface subdivision. In our paper [15], we develop the analogous theory, along with various examples for two newly introduced splits, called square-root 5 and square-root 7 splits.

Another attempt to construct smooth bivariate spline surface representation of scattered data is studied in [17], where an optimality criterion is derived to place sample points within an arbitrary triangulation, corresponding to the given scattered data set, and an efficient local algorithm associated with this criterion is derived for generating a C^1 bivariate spline surface to interpolate the discrete scattered data. However, no basis functions are used in this approach, and the method is therefore not multi-level.

In a more recent development, the interpolating subdivision templates derived in our papers [6, 7, 16] are being applied to generate data-dependent basis functions for interpolation of arbitrary data using the multi-level architecture of subdivisions and without carrying the data on any mesh. Various complex terrain elevation data sets are used as test examples for our current study.

5. Wavelet analysis and spline-wavelet tight frames [2, 3, 8, 10, 12, 13]

As is well known, the most intimate and useful companions of refinable functions or function vectors are their associated wavelets. Unfortunately, compactly supported refinable spline functions cannot be orthogonal. Furthermore, their spline duals have infinite support, and compactly supported bi-orthogonal duals do not have explicit formulation. A natural and practical approach is to replace orthogonality by the notion of tight frames. Indeed, tight frames preserve the Parseval (series) representation formulas enjoyed by orthonormal wavelets, with coefficients of the series representations given by discrete wavelet transforms (DWT). Unfortunately, for B-splines of order k > 1 that locally preserve polynomials of order k (or degree k -1), at least one set of the DWT has only first-order vanishing moments, meaning that even linear polynomial terms are not be annihilated by these DWT coefficients. The notion of vanishing moment recovery (VMR) was therefore introduced in our research program supported by our previous ARO grant to preserve the maximum order of vanishing moments of the DWT. In our paper [2], we have developed a parallel theory for the non-stationary setting on unbounded intervals. As opposed to our previous paper concerning the non-stationary setting on bounded intervals where an algebraic approach was employed, we need to take an analytical approach for the unbounded interval setting, and for this paper [2], we employ the kernel methods. It is interesting to point out that the previous result on bounded intervals cannot be derived from the results in [2] on unbounded intervals, by keeping the interior wavelet basis functions while modifying the ones with supports containing the end-points, as in the construction of orthonornal, semi-orthogonal, and bi-orthogonal wavelets, without loss of either the tight frame condition or the number of vanishing moments for some boundary wavelets.

The key property of wavelets as an analysis tool is their vanishing moments. This property can be translated to the efficiency of the wavelet considered as a band-pass filter, in separating the band-pass efficiency away from the low-frequency band. In [10], a systematic method is introduced to construct multi-wavelets of higher dimensions to increase the order of vanishing moments (for all components of the multi-wavelet), starting from any of the (one-dimensional) Daubechies orthonormal wavelets. The orders of vanishing moments (for all components) are always guaranteed to increase with each increase of the (vector) dimensions by one.

In application of multi-wavelets to process scalar-valued data without creating artificial higher dimensional components of the data set, the simplest way is to group the scalar data as vectors with the same dimension as the multi-wavelet. Unfortunately, this often results in very undesirable artifacts. On the other hand, lifting each data point to the same higher dimensional space as that of the multi-wavelet by introducing artificial data components is certainly a bad practice for data compression, since it increases the data file size before the compression algorithms are applied. The natural and commonly used procedure is to pre-condition the data sets. However, such methods are in general not efficient, as the more effective ones have to depend on the data contents. The notion of "balanced multi-wavelets" was introduced in the literature some 10 years ago. In our paper [3], we put the proposed definition in a firm ground, extend the definition to the multivariate setting, introduce the concept of balanced centers, and derive a set of useful equivalent conditions to facilitate the construction balanced multi-wavelets, as well as the study of their analysis capability.

The capability of constructing multivariate wavelets on arbitrary triangulations (for the bivariate study, and for the extension to simplices in higher dimensions) is still in its infancy. In fact, even the current results on wavelets of bivariate piecewise linear splines are not satisfactory, due to either the need of very support or the lack of vanishing moment of the second order. On the other hand, the most powerful approach to constructing univariate wavelets is carried out in the Fourier domain. In our paper [13], we derive explicit formulas of the Fourier transform of Bernstein-Bezier polynomials on simplices, in order to provide some effective tool for the future study of multivariate spline wavelets on simplices. Some illustrative examples are given in this paper.

Our papers [8, 12] are highly theoretical and very general. In [8], we give a comprehensive study of affine (or wavelet) frame decompositions and shift-invariant spaces, including derivation of certain complete characterizations of all closed subspaces and "angular distances" between any two of them. The notion of angles is new, and generalizes the classification and identification of all orthogonal subspaces. Affine (or wavelet) frames can be constructed by means of "over-sampling", a notion introduced in our earlier paper about 15 years ago. In this first paper of a new research area, we considered wavelets with dilation factor = 2, and proved that over-sampled frames obtained by over-sampling of a given tight frame (such as an orthonnormal basis), by using over-sampling rate = n, remain to be tight frames, if n is an odd integer. Later, we proved that this result is sharp, and generalize certain directions to include matrix dilations. In our recent paper [12], we give a comprehensive study of the entire problem

area, including derivation of complete characterizations of all tight over-sampled affine frame systems, as well as all integer matrices that can be used as over-sampling rates to preserve frame tightness.

6. Development of data processing tools based on minimum-energy criteria [14]

The commonly used minimum-energy approach is adopted to derive variational algorithms based on given data in the physical domain, but with internal energy functional defined on the wavelet domain in our paper [14]. A general theory is developed in this paper, including the use of Euler-Lagrange equations, steepest decent, diffusion and diffusion-convection partial differential equations (PDE's), and various choices of diffusion conductivity functions. More importantly, some appropriate selections of internal energy density functions for the discrete wavelet domain immediately give rise to the most popular wavelet shrinkage methods, called hard thresholding and soft thresholding. A significant impact of our minimum-energy approach to arrive at wavelet thresholding is that simple formulas for computing precise thresholding parameters are naturally generated from the optimization criteria. In comparison with the commonly used oracle approach in the literature, as introduced by Donoho and Johnstone, we have noticeably better noise reduction results, both for removing iid noise from one-dimensional noisy data and Gaussian white noise for image noise removal. Typical examples in the literature are used for these comparisons in the paper [14].

7. Applications to computer graphics and image restoration [1, 5, 11, 14, 18]

Subdivision curves and surfaces are easy to render but difficult to edit. Fortunately, with the introduction of wavelets, it seems to be feasible to construct efficient and effective editing tools. An enormous amount of research has been carried out over the past 15 years in the search of such algorithms and techniques. The most popular ones are based on the semi-orthogonal spline wavelets we introduced about 16 years ago (and described in details in my two books: An Introduction to Wavelets, Academic Press, 1992; and Wavelets: A Mathematical Tool for Signal Analysis, SIAM, 1997). However, as described in Section 4 above, semi-orthogonal spline wavelets do not have compactly supported spline duals, and hence do not support finite linear algorithms required repeatedly for the editing process.

In our paper [5], we modify the tight frame of cubic spline wavelets on the bounded interval from our previous work to satisfy the editing requirement, and apply the modified tight frame to introduce an interactive system for generating artistic sketches from image contents, based on the livewire contour tracing paradigm. The construction of livewire stroke maps of sketch details at different multi-resolution spline levels leads to user-guided stokes, automatically locked on to the target contours, to facilitate fast and accurate sketch drawing. The target contours are classified as "outlines" and "interior flow" for the development of two respective livewire techniques based on extended graph structure and vector flow field.

In the area of processing digital image data, the problem of converting photographs to sketches and paintings is attracting a lot of attention recently. In our paper [11], we derive a comprehensive scheme for automatically generating a broad class of artistic illustrations from photographs by using strokes as major building blocks. The system introduced in this work optimizes stroke attributes subject to the desired rendering style, while enabling adaptive control of the abstraction level for each pixel. Our outline construction paradigm, called "edge painting" in this paper, is aimed at producing outline maps and sketches adaptively. This is accomplished by extending the bilateral filter approach (introduced by Tomasi and Manduchi in 1998 for image noise removal) to adaptive filtering. Given the outlines and direction maps, the system creates the final illustration via selecting the representative colors, setting the style parameters, and optimizing the stroke attributes based on simulated annealing. Experimental results show that our scheme facilitates automatic production of artistic illustrations for a wide range of rendering styles.

Our papers [1, 14] are concerned with digital image restoration. As described in the previous section, consideration of certain appropriate minimum-energy criteria leads to non-linear PDE models, including those that describe anisotropic diffusion processes. While the popular bilateral filter mentioned above also describes some kind of diffusion process and is effective in reducing Gaussian white noise, it is certainly not effective in removing random noise in image restoration. In our paper [1], we introduce another filter component which we call ROAD filtering. When the ROAD filter is combined with the spatial and radiometric filter components of the bilateral filter to yield a trilateral filter, as introduced in [1], mixtures of Gaussian white noise and random noise can be removed from contaminated digital images effectively.

For computer graphics, while traditional study has mainly pursued sheer photorealism, the non-photorealistic rendering (NPR) paradigm has recently emerged as a better alternative in many applications involving visual communication, feature extraction and simplification, data compression, or aesthetic visual expression. In our recent paper [18], we present a NPR rendering technique that automatically generates a line drawing from a photograph. In this paper, we aim at extracting a set of coherent, smooth, and stylistic lines that effectively capture and convey important shapes in the image. We first develop a novel method for constructing a smooth direction field that preserves the flow of the salient image features, according to certain diffusion process, such as bilateral filtering. We then introduce the notion of flow-guided anisotropic filtering for detecting highly coherent lines while suppressing noise. Our method is simple and easy to implement. A variety of experimental results are presented to show the effectiveness of our method in producing self-contained, high-quality line illustrations.

One of our ultimate goals in processing image-based data to NPR rendering is to integrate the approach introduced in [18] along with the one-ring matrix interpolating subdivision schemes, discussed in Sections 2 and 3 above, to generate interpolating surfaces with embedded realistic and/or artistic features, such as textures for animation movie production, and more importantly, certain desirable attributes, such as ridges and

roughness, for complex terrain modeling and elevation data interpolation. This should have significant positive impact both to the entertainment industry as well as to geospatial applications.

Research papers published under the sponsorship of this grant

Journal papers published in 2005

(1) A universal noise removal algorithm with an impulse detector (with R. Garnett, T. Huegerich, W. He), *IEEE Trans. Image Processing*,
Vol. 14, No.11 (2005), 1747--1754.

(2) Nonstationary tight wavelet frames II: unbounded intervals (with W. He and J. Stoeckler), *Appl. and Comp.Harmonic Anal.,*
Vol. 18 (2005), 25--66.

(3) Balanced multi-wavelets in R^s (with Q.T. Jiang), *Math. of Computation*, Vol. 74 (2005), 1323--1344.

(4) Matrix-valued symmetric templates for interpolatory surface subdivisions I: Regular vertices (with Q.T. Jiang), *Appl. and Comp. Harmonic Anal*.,
Vol. 19 (2005), 303--339.

(5) Iterative sketch generation (with W. He, H. Kang, and U. Chakraborty), *The Visual Computer*, Vol. 21, No. 9 (2005), 812--830.

(6) Refinable bivariate C²-splines for multi-level data representation and surface display (with Q.T. Jiang), *Math of Computation,*
Vol. 74 (2005), 1369--1390.

Journal papers published in 2006

(7) Refinable bivariate quartic and quintic C²-splines for quadrilateral subdivisions (with Q.T. Jiang), *J. of Comp. and Appl. Math.*,
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(8) Affine frame decompositions and shift-invariant spaces (with Q. Sun), *Appl. Comp. Harmonic Anal.,* Vol. 20 (2006), 74--107.

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